Updates to the User's Manual and Statistical Theory Appendix for *RCT-YES* Version 1.3

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November 2019

RCT-YES **was developed by Mathematica Policy Research, Inc. with funding from the Institute of Education Sciences under Contract ED-IES-12-C-0057 with Decision Information Resources, Inc.**

NATIONAL CENTER FOR EDUCATION EVALUATION AND REGIONAL ASSISTANCE

Institute of Education Sciences
U.S Department of Education

This manual discusses updates to the *RCT-YES* software for Version 1.3. It serves as a supplement to the more detailed May 2016 and January 2018 *RCT-YES* User's Manuals and Statistical Theory Appendixes (all found at www.rct-yes.com).

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Introduction

The free *RCT-YES* software (www.rct-yes.com) estimates and reports average treatment effects for evaluations of interventions, programs, and policies using randomized controlled trial designs (RCTs) or quasi-experimental designs (QEDs) with comparison groups. The software is applicable to a wide range of evaluation designs used in social policy and related research. The methods underlying the software are based on a new design-based statistical theory that has important advantages over traditional model-based methods used in social policy research (Schochet, 2016; 2018). The software is user friendly with no knowledge of computer programming required. The software reports study findings in formatted tables and graphs that meet industry standards, and conform to What Works Clearinghouse evidence reviews (Scher and Cole, 2017).

RCT-YES Version 1.0 was released in May 2016 with associated documentation available at www.rct-yes.com. Version 1.1 was released in June 2016 to fix minor program bugs. Version 1.2 was released in January 2018 with important new features implemented in response to user feedback. The key new feature was that the software can now accommodate designs with more than two research groups (multi-armed designs). Version 1.3 provides a slightly updated variance estimator for random (super-population) block designs where the study blocks (such as sites) are assumed to be sampled from a broader block population.

This brief note discusses the statistical theory updates in Version 1.3. The new version does not involve any changes to the program inputs or data dictionary, but involves a slight change to the variance estimators for random block designs for the super-population model.

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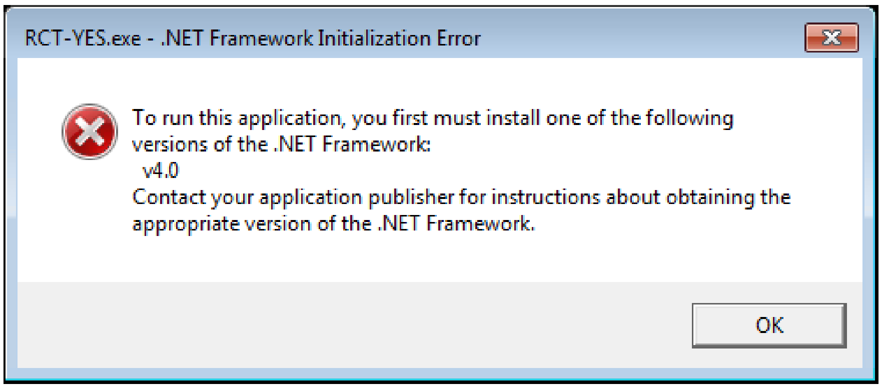
Version 1.3 Updates: November 2018

RCT-YES Version 1.3 was released in April 2018 and updates Version 1.2 which was released in January 2018.

If your installed version of .NET is outdated, you will receive the following error when you try to download *RCT-YES* Version 1.3.



If you receive the above error message, please download and run the installer file for [.NET 4.8] (click "Run" when prompted). After the .NET 4.8 installation completes, you should be able to download *RCT-YES* Version 1.3.

A. Updates to Program Inputs

Version 1.3 does not involve any changes to the program inputs or the data dictionary.

B. Updates to Running the Program

Version 1.3 involves no updates on the process for running the program and generating the output files and graphs.

C. Updates to Statistical Theory

1. Variance estimators have been slightly adjusted for random block designs (Designs 2 and 4) for the super-population model

In random block designs, the study blocks (such as sites) are assumed to be randomly sampled from a super-population of blocks, so that the study results are assumed to generalize to this broader population. For non-clustered designs, random block designs essentially involve three stages of sampling: (1) blocks, (2) individuals within blocks, and (3) randomization of individuals to the treatment and control groups. For clustered designs, random block designs involve four stages of sampling: (1) blocks, (2) clusters within blocks, (3) randomization of clusters, and (4) individuals within clusters.

In previous software versions, *RCT-YES* estimated variances for random block designs (specified using the options SUPER_POP=1 and CATE_UATE=0 or 2) using the following variance formula where, for illustration, we assume a simple differences-in-means estimator and a non-clustered design (see Equation (6.25) on page 83 of the Statistical Theory Appendix [Schochet, 2016]):

$$(1) \quad As\hat{y}Var(\hat{\beta}_{PATE}) = \frac{1}{(h-1)h\overline{w}^2}\sum_{b=1}^{h}(w_b\hat{\beta}_{b,PATE} - \overline{w}\hat{\beta}_{PATE})^2 \ .$$

In this expression, $\hat{\beta}_{PATE} = \sum_{b=1}^{h} w_b\hat{\beta}_{b,PATE} / \sum_{b=1}^{h} w_b$ is the pooled impact estimate across the h blocks, $\hat{\beta}_{b,PATE}$ is the impact estimate in block b, w_b is the block-specific weight, and $\overline{w} = \sum_{b=1}^{h} w_b / h$ is the average block weight.

As shown in Lemma 6.2 in Schochet (2016), conditional on the weights, the variance estimator in Equation (1) is consistent for the true asymptotic variance of $\hat{\beta}_{PATE}$:

$$(2) \ AsyVar_{RIB}(\hat{\beta}_{PATE}) = \frac{1}{hE_B(w_b)^2}\left[E_B[w_b^2(\frac{\sigma_{Tb}^2}{n_{Tb}} + \frac{\sigma_{Cb}^2}{n_{Cb}})] + Var_B(w_b(\mu_{Tb} - \mu_{Cb}))\right],$$

where expectations are sequentially taken over the randomization distribution (R) in each block, the super-population of students (I) in each block, and finally over the super-population of blocks (B). In this expression, n_{Tb} is the number of treatment individuals, n_{Cb} is the number of control indiviuals, μ_{Tb} and μ_{Cb} are mean potential outcomes in I, and σ_{Tb}^2 and σ_{Cb}^2 are corresponding super-population variances.

The variance estimator in Equation (1) assumes that the block weights and impacts are independent. However, an *RCT-YES* user pointed out that this assumption might not realistic in some real-world applications, where the block weights and impacts could be correlated. For example, in a multi-site trial, site-level impacts might be larger (or smaller) in more populous sites with larger weights than in less populous sites.

To adjust for these potential correlations, *RCT-YES* now uses the following variance estimator:

$$(3) \quad As\hat{y}Var(\hat{\beta}_{PATE}) = \frac{1}{(h-1)h\overline{w}^2}\sum_{b=1}^{h} w_b^2(\hat{\beta}_{b,PATE} - \hat{\beta}_{PATE})^2,$$

which treats the weights slightly differently than in Equation (1) by replacing $\overline{w}$ with w_b in the parentheses in the numerator. Thus, the weights now appear outside the parentheses as squared terms rather than inside the parentheses.

The consistency of the variance estimator in Equation (3) for the asymptotic PATE variance in Equation (2) can be established using results in Cochran (1977; Theorem 11.2 and pages 300-305) on the variance of ratio estimators for population means (and totals) for two-stage clustered designs. To provide intuition on this result, note that Cochran considers variance estimation for ratio estimators of the form $\hat{\bar{y}}_{CR} = \sum_{b=1}^{h} w_b \bar{y}_{Cb} / \sum_{b=1}^{h} w_b$, where $\bar{y}_{Cb}$ is the observed mean outcome, say for the control group, for cluster b. Cochran considers designs with sampling from finite universes, but we can extend his results to consider sampling from infinite populations at both stages. In this context, Cochran's results (Equation 11.30) imply that the variance estimator,

$$(4) \quad As\hat{y}Var(\hat{\bar{y}}_{CR}) = \frac{1}{(h-1)h\bar{w}^2} \sum_{b=1}^{h} w_b^2 (\bar{y}_{Cb} - \hat{\bar{y}}_{CR})^2,$$

is a consistent estimator for the true asymptotic variance of $\hat{\bar{y}}_{CR}$:

$$(5) \quad AsyVar_{IB}(\hat{\bar{y}}_{CR}) = \frac{1}{hE_B(w_b)^2} \left[E_B[w_b^2 (\frac{\sigma_{Cb}^2}{n_{Cb}})] + Var_B(w_b \mu_{Cb}) \right].$$

In this expression, the first term pertains to the variation of individual outcomes within clusters while the second term pertains to the variation of mean outcomes between clusters in the super-population.

We can now extend these results to random block designs by viewing blocks as clusters, and viewing the treatment and control group samples as separate random samples from common super-populations in each block. The outcomes of the treatment and control groups might be correlated within a block due to a common block effect (which can improve precision), but the two samples are drawn independently. Thus, we can establish the consistency of Equation (3) by replacing $\bar{y}_{Cb}$ with $\hat{\beta}_{b,PATE}$ and $\hat{\bar{y}}_{CR}$ with $\hat{\beta}_{PATE}$ in Equations (4) and (5). Intuitively, for multi-stage clustered designs, parallel variance formulas apply for estimating means and estimating the difference between two correlated means measured from the same clusters.

The same adjustments to the variance formulas for parallel random block designs were made in *RCT-YES* for models with covariates, clustered designs, subgroup analyses, and baseline equivalency analyses.

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References

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Schochet, P.Z. (2018). *Updates to the user's manual and statistical theory appendix for RCT-YES Versions 1.2 and 1.1*. https://www.rct-yes.com/Content/PDF/RCT_YES_User's_Manual_Update_for_Version_1.2.pdf

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